MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

9 MARCH 2019
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

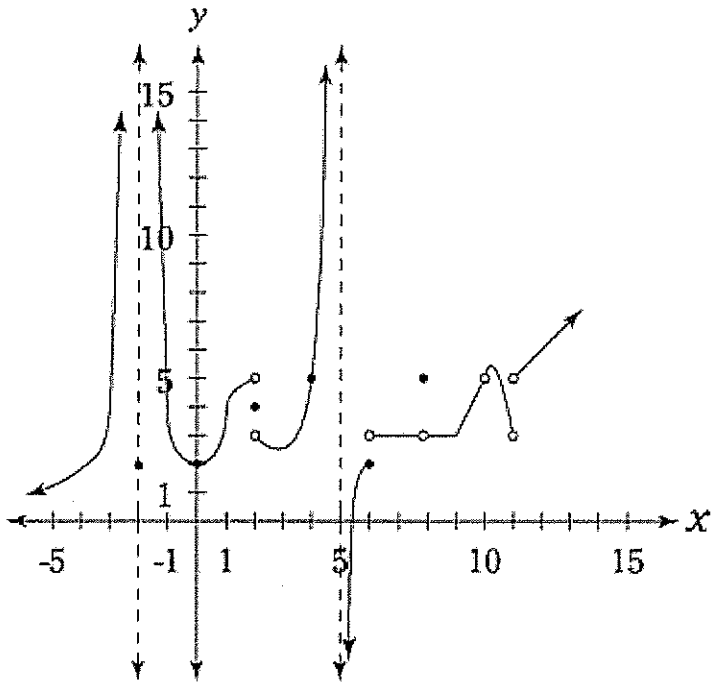
1. This Question paper consists of 2 pages excluding the cover page and appendix.

2. Answer **all FIVE** questions. Each question carries equal marks and the distribution of the marks is given.

3. Write all your answers in the Answer Booklet provided.

Question 1 [10 marks]

a.



i. Based on the graph above, identify the x-values where the requirements for the existence of a limit is satisfied and what are the limit at those x values. (4 marks)

ii. At what values of x where all the three requirements for continuity are met? (1 mark)

b. Find the following limits.

i. $\lim_{x\to-\infty}\dfrac{1+x+5x^2}{-3+x-3x^3}$ (2 marks)

ii. $\lim_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$ (3 marks)

Question 2 [10 marks]

a. Find the derivative of the following functions.

i. $R(w)=\dfrac{3w+w^4}{2w^2+1}$ (2 marks)

ii. $xy^3-2x^4y=1-\ln y$ (3 marks)

Continued...

b. Evaluate the following integrals.

i. $\int_1^2 \frac{2x^5 - x + 3}{x^2} dx$ (2 marks)

ii. $\int x^3 \ln 5x dx$ (Hint: Use integration by parts method) (3 marks)

Question 3 [10 marks]

a. Given $f(x) = x^5 - 5x^4 + 5x^3$.

i. Identify the critical numbers and classify these critical points as local maximum, local minimum or neither. (3 marks)

ii. Determine the intervals on which the function is concave up and concave down and the x-coordinate of the inflection point(s). (3 marks)

b. A farmer has 2400 meter of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area? (4 marks)

Question 4 [10 marks]

a. i. Sketch the graph $y = x^2$ and $y = x + 6$. Shaded the region enclosed by these curve and line and also the lines $x = 0$ and $x = 5$. (1 mark)

ii. Find the area of the region enclosed by the curve and the line. (4 marks)

b. Find the volume of a solid generated by rotating the region bounded by $y = x^2 + 2$ and $y = x + 4$ about the x-axis. (5 marks)

Question 5 [10 marks]

a. Find the general solution for the first order differential equation $\frac{dy}{dx} + \frac{4}{x} y = x$. (5marks)

b. Find the unique solution for the second order differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 0 \text{ where } y(0) = 2 \text{ and } y'(0) = -1$$

(5 marks)

End of Paper

APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}$; $x > 0$

B. Basic Integration Formulas

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$